Reg. No. : [ ][ ][ ][ ][ ][ ][ ][ ][ ][ ][ ][ ]

Question Paper Code : 53246

B.E./B.Tech. DEGREE EXAMINATIONS, APRIL/MAY 2019.

Second Semester

Civil Engineering

MA 6251 — MATHEMATICS – II

(Common to All Branches Except Marine Engineering)

(Regulation 2013)

Time : Three hours   Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. What is the greatest rate of increase of $\phi = xyz^2$ at (1, 0, 3)?

2. State Gauss - Divergence theorem.

3. Solve $(D^3 + D^2 - D - 1)y = 0$.

4. Transform the equation $(x^3D^3 + 9x^2D^2 + 18xD + 6)y = x^2$ in to a linear equation with constant coefficients.

5. Find the Laplace transform of unit step function.

6. State the initial value theorem under Laplace transforms.

7. Define analytic function.

8. Find the image of the circle $|z| = 2$ under the transformation $w = z + 2 + 3i$.

9. Identify the type of singularity of $f(z) = (1 - \cos z)/z$ at $z = 0$.

10. Find the residue of $f(z) = \dfrac{z^2}{(z-1)(z-2)}$ at $z = 2$.

PART B — (5 × 16 = 80 marks)

11. (a) (i) What is the angle between the surfaces $x^2 + y^2 + z^2 = 9$ and $x^2 + y^2 - z - 3 = 0$ at the point $(2, -1, 2)$? (8)

(ii) Using Stoke's theorem, evaluate $\int_C xydx + xy^2 dy$ taking C is a square bounded by the lines $x = 1$, $x = -1$, $y = 1$, $y = -1$. (8)

Or

(b) (i) Show that $\vec{F} = (z^2 + 2x + 3y)\vec{i} + (3x + 2y + z)\vec{j} + (y + 2\ xz)\ \vec{k}$ is irrotational and hence find the corresponding potential function ϕ. (8)

(ii) Using Gauss Divergence theorem, evaluate $\iint curl\ \vec{F} \cdot \vec{n}\ dS$ where $\vec{F} = 4xz\vec{i} - y^2\vec{j} + yz\vec{k}$ over the cube bounded by $x = 0$, $x = 1$, $y = 0$, $y = 1$, $z = 0$, $z = 1$. (8)

12. (a) (i) Solve $(D^2 - 4D + 3)\ y = \sin 3x + x^2$. (8)

(ii) Using method of variation of parameters, solve $(D^2 + 1)\ y = \sec x$. (8)

Or

(b) (i) Solve $\frac{dx}{dt} + \frac{dy}{dt} + 5x + 7y = 2e^{2t}$, $2\frac{dx}{dt} + 3\frac{dy}{dt} + x + y = e^t$. (8)

(ii) Solve $(x^2D^2 - 3xD + 4)y = x^2$. (8)

13. (a) (i) Find the Laplace transform of $t^2 e^t \sin 4t$. (8)

(ii) Using Convolution theorem, find the inverse Laplace transform of $\frac{1}{(s+1)(s^2+1)}$. (8)

Or

(b) (i) Find the Laplace transform of $f(t)=\begin{cases} t : 0<t<1 \\ 0 : 1<t<2 \end{cases}$ and $f(t+2)=f(t)$. (8)

(ii) Solve $y''+4y'+8y=1$, $y(0)=0$, $y'(0)=1$ by using Laplace transforms. (8)

14. (a) (i) Find the analytic function $f(z)=u+iv$ if $u=e^{x^2-y^2}\cos 2xy$. Hence find v. (8)

(ii) Find the image in the w plane of the region of the z plane bounded by the straight lines $x=1$, $y=1$, $x+y=1$ under the transformation $w=z^2$. (8)

Or

(b) (i) If $w=u(x,y)+iv(x,y)$ is an analytic function, the curves of the family $u(x,y)=a$ and the curves of the family $v(x,y)=b$ cut orthogonally, where a and b are constants. (8)

(ii) Find the bilinear transformation which maps the points $z=0$, $z=1$, $z=\infty$ in to the points $w=i$, $w=1$, $w=-i$ respectively. (8)

15. (a) (i) Use Cauchy's integral formula to evaluate $\int_C \frac{z+1}{z^3-2z^2}dz$, where C is the circle $C:|z-2-i|=2$. (8)

(ii) Find the Laurent's series of $f(z)=\frac{z}{(z-1)(z-2)}$ valid in the region $|z+2|<3$ and $3<|z+2|<4$. (8)

Or

(b) Evaluate $\int_{-\infty}^{\infty} \frac{x^2}{(x^2+a^2)(x^2+b^2)}dx$, $a>b>0$, by using contour integration. (16)